Numerical modelling of nonlinear 3- and 4-wave interactions in spectral sea-state models in finite and variable water depth, with effects of an ambient ocean current

Alessandro Guerri[1,2], **Michel Benoit**[1,2], **Thierry Fouquet**[1], **Maria Teles**[1]

[1] EDF R&D - Laboratoire National d'Hydraulique et Environnement (LNHE), 6 quai Watier, 78400 Chatou.

[2] Laboratoire d'Hydraulique Saint-Venant (Ecole des Ponts, EDF R&D), 6 quai Watier, 78400 Chatou.

Context This thesis aims to improve knowledge of the physics and numerical modelling of nonlinear wave-wave interactions from the oceanic domain to the coastal zone, considering the effects of an ambient marine current. Depending on the relative water depth, the dominant interactions occur between 4 or 3 waves; they play a fundamental role in the dynamics of the wave field and control the shape of directional energy spectrum of the sea state (cf. Cavaleri et al. (2007), for example).

These modifications to the spectrum, corresponding to the evolution of wave trains in the physical domain (shape and height of wave troughs and crests, horizontal and vertical asymmetries of the wave profile, presence of long waves, etc.), are fundamental for applications in the maritime and coastal domain.

More accurate prediction methods are essential to control metocean conditions for conducting operations at sea, designing offshore installations and coastal structures, etc.

Mathematical models

TOMAWAC model (Benoit *et al.*, 1996) of the Telemac-Mascaret modelling suite (http://www.opentelemac.org/):

- Balance equation for wave action N:

$$\frac{\partial N}{\partial t}+\frac{\partial(\dot{x}N)}{\partial x}+\frac{\partial(\dot{y}N)}{\partial y}+\frac{\partial(\dot{k}_x N)}{\partial k_x}+\frac{\partial(\dot{k}_y N)}{\partial k_y}=Q(k_x,k_y,x,y,t)$$

- Mathematical models for energy input, transfer and dissipation terms Q, including (among others):
  - Q_{in}: wind-driven wave generation
  - Q_{ds}: whitecapping-induced energy dissipation
  - Q_{nl}: non-linear quadruplet interactions
  - Q_{tr}: non-linear triad interactions

Numerical models for Q_{nl}

Focusing on non-linear quadruplet interactions term Q_{nl}, expressed by (Hasselmann, 1962):

$$Q_{nl}^{exact}=\iiint \sigma_4.G.\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_4\right)\delta\left(\sigma_1+\sigma_2-\sigma_3-\sigma_4\right)$$

$$\left[\frac{F(\boldsymbol{k}_1)}{\sigma_1}\frac{F(\boldsymbol{k}_2)}{\sigma_2}\left(\frac{F(\boldsymbol{k}_3)}{\sigma_3}+\frac{F(\boldsymbol{k}_4)}{\sigma_4}\right)-\frac{F(\boldsymbol{k}_3)}{\sigma_3}\frac{F(\boldsymbol{k}_4)}{\sigma_4}\left(\frac{F(\boldsymbol{k}_1)}{\sigma_1}+\frac{F(\boldsymbol{k}_2)}{\sigma_2}\right)\right]d\boldsymbol{k}_1 d\boldsymbol{k}_2 d\boldsymbol{k}_3$$

- Approximate methods (among others):
  - DIA, Discrete Interaction Approximation
  - mDIA, multiple DIA
- Quasi-exact methods (among others):
  - GQM, Gaussian Quadrature Method

Objectives (first part)

- Understanding the role of quadruplet interactions in sea state dynamics through quasi-exact GQM method.
- Comparison of GQM calculations with other methods (DIA, mDIA).
- Extension of GQM applicability to finite water depth conditions.
- Taking into consideration the effect of ambient current.

Quadruplet interactions role in sea state dynamics (GQM simulations)

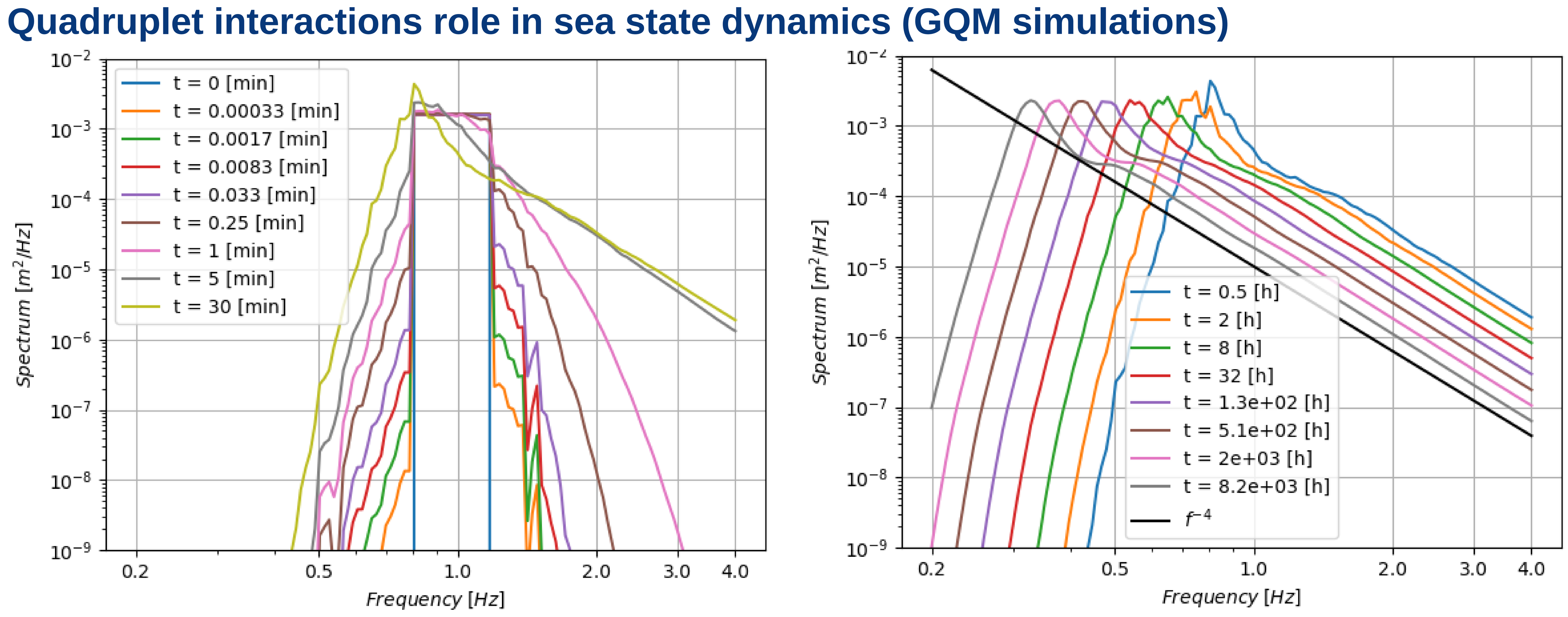

FIG. 1 - EVOLUTION OF FREQUENCY SPECTRUM UNDER THE ONLY EFFECT OF Qnl, BEGINNING FROM STEP-SHAPED

Comparison of GQM calculations with DIA (example of angular spreading)

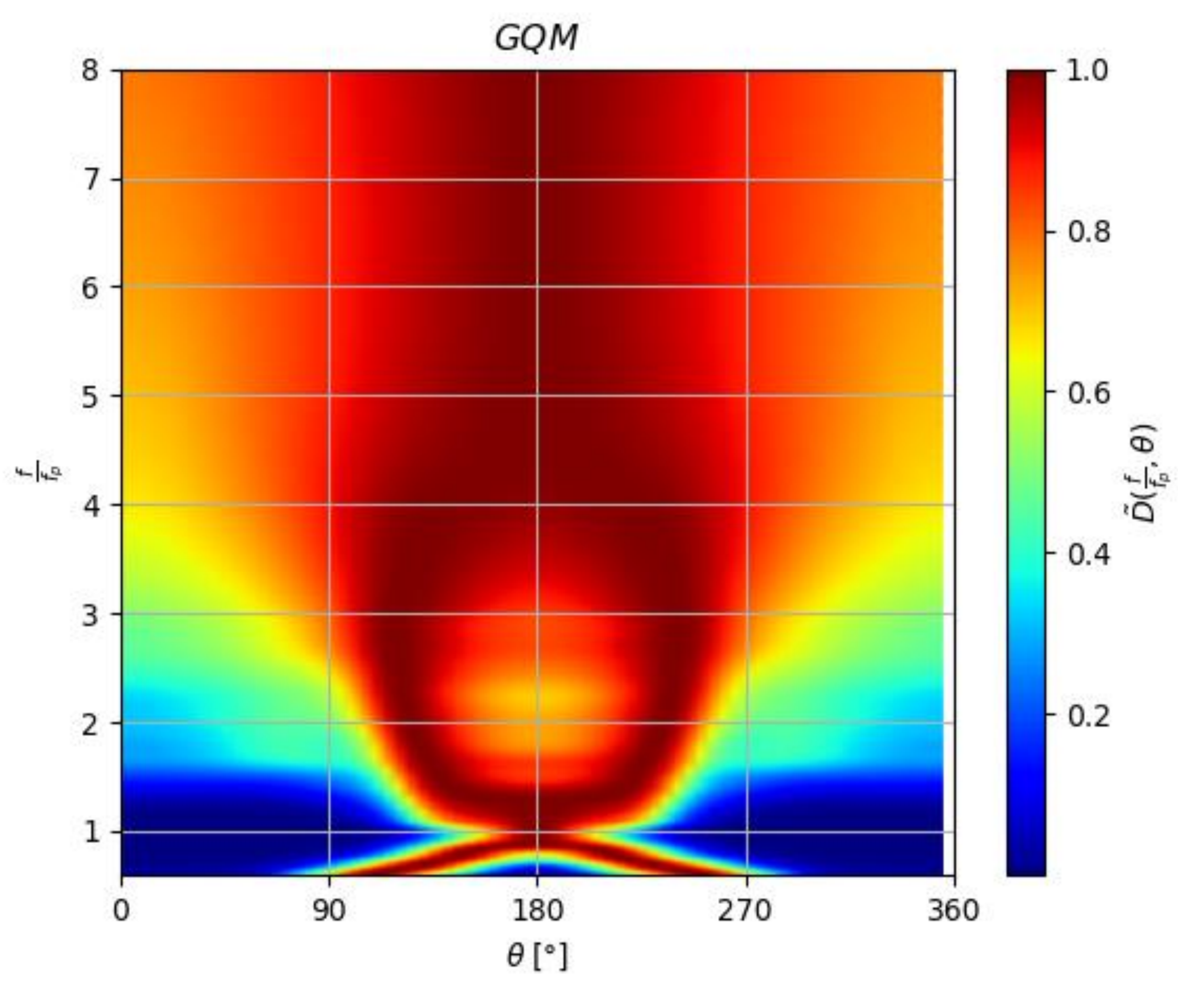

FIG. 2 - FINAL SPREADING OBTAINED WITH GQM. EVIDENT BIMODAL STRUCTURE

- Very long simulation run beginning from an initial spectrum defined by:

$$F(f,\theta)=E(f)D(\theta)$$

with $E(f)$ JONSWAP ($\gamma=3$, $f_p=1\,Hz$ and $D(\theta)$ broad cos^{2s} function)

- Spectrum evolution under only interacting quadruplets effect
- Same simulation run with GQM and DIA
- Focus on the final spreading function $\widetilde{D}$, normalized as:

$$\widetilde{D}\left({}^{f}\!/_{f_p},\theta\right)=\frac{D\left({}^{f}\!/_{f_p},\theta\right)}{max_\theta\left(D\left({}^{f}\!/_{f_p},\theta\right)\right)}$$

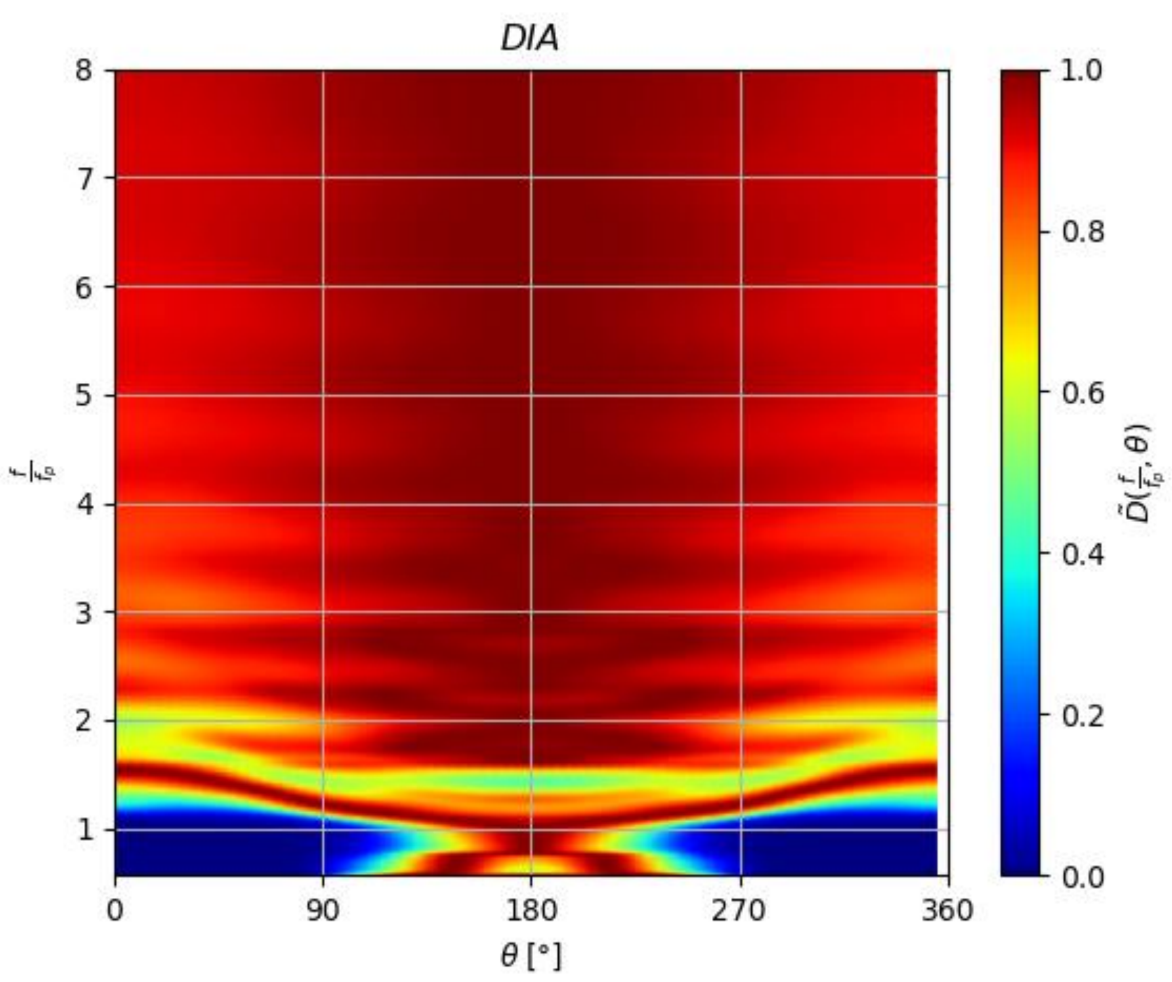

FIG. 3 - FINAL SPREADING OBTAINED WITH DIA. BIMODALITY IS LESS MARKED AND DIFFERENT FROM GQM

Conclusions

- Key role of 4-waves interactions in sea state dynamics:
  - Formation of spectral peak and f^{-4} tail
  - Once near equilibrium spectral shape is reached, shifting of the peak towards lower frequencies
- Discrepancies between quasi-exact GQM and approximate DIA methods

Next steps

- Optimization of GQM CPU time (elimination of least important quadruplet configurations, optimal number of integration points, …)
- Extension of GQM algorithm to finite water depth
- Taking into consideration the effect of ambient current

References

S.I. Badulin, A.N. Pushkarev, D. Resio, V.E. Zakharov, Self-similarity of wind-driven seas, *Nonlinear Processes in Geophysics*, **12**, 894-945 (2005).

M. Benoit, M. Marcos, F. Becq, Development of a third generation shallow-water wave model with unstructured spatial meshing, *Proc. 25-th Int. Conf. On Coast. Eng. (ICCE'1996), Orlando (USA), ASCE*, 465-478 (1996).

L. Cavalieri *et al.*, Wave modelling – The state of art, *Progress in Oceanography*, **75**, 603-674 (2007).

E. Gagnaire-Renou, M. Benoit, P. Forget, Ocean wave spectrum properties as derived from quasi-exact computations of nonlinear wave-wave interactions, *Journal of Geophysical Research C (Oceans)*, **115**(C12), C12058 (2010).

K. Hasselmann, On the non-linear energy transfer in a gravity-wave spectrum. Part I: General theory, *Journal of Fluid Mechanics*, **12**, 481-500 (1962).

I.V. Lavrenov, Effect of wind-wave parameter fluctuation on the nonlinear spectrum evolution, *Journal of Physical Oceanography*, **31**, 861-873 (2001).